A counter-intuitive mechanics and traffic flow problem solved by mathematics

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Question

Can adding a road to a congested network cause more traffic?

What happens?

Definition

A congested network is a network in which, for at least one segment, the cost of travel (time) strictly increases with increasing traffic flow.

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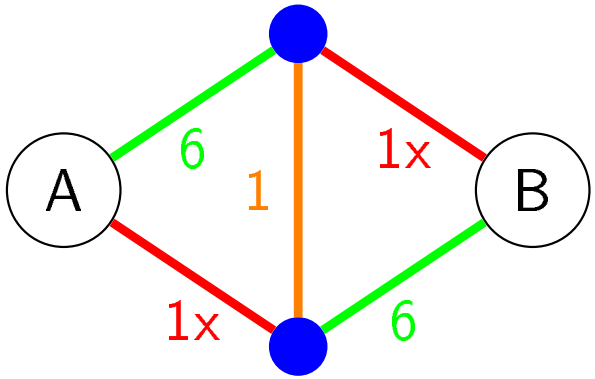
Paradox (Dietrich Braess, 1968)

There exists congested transportation networks such that, if a link is added and if all individuals seek their best possible route, then the cost of travel for all individuals is higher than before the route was added.

Photo: Renate Schmid

What happens?

With the (orange) highway:



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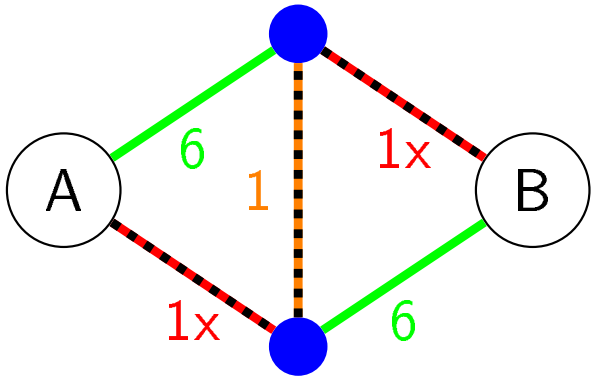

Minimal path: 9 minutes

What happens?

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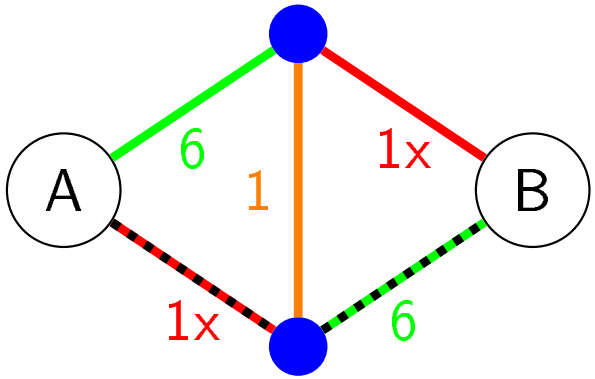

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Alternative paths:

- 10 minutes

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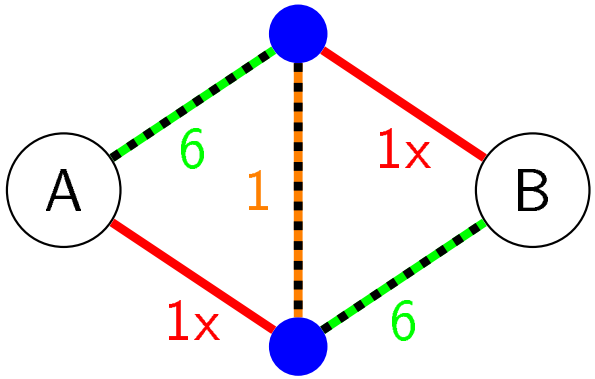

Minimal path: 9 minutes

Alternative paths:

- 10 minutes
- 13 minutes

What happens?

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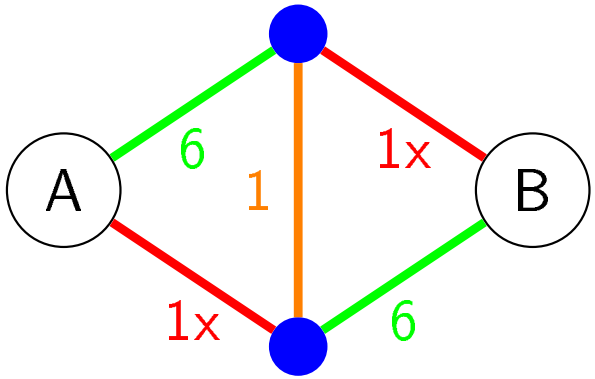
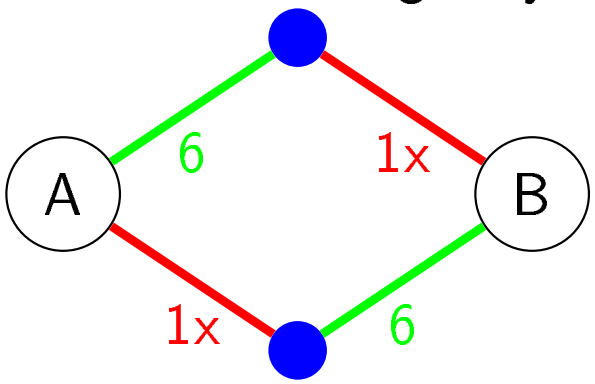


Without the highway:



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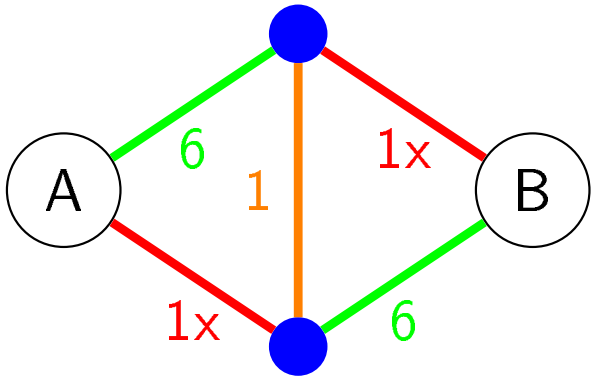

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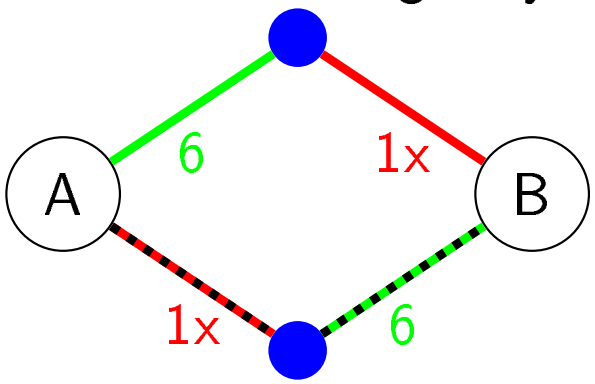

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Minimal path: 8 minutes
(2 cars each side)

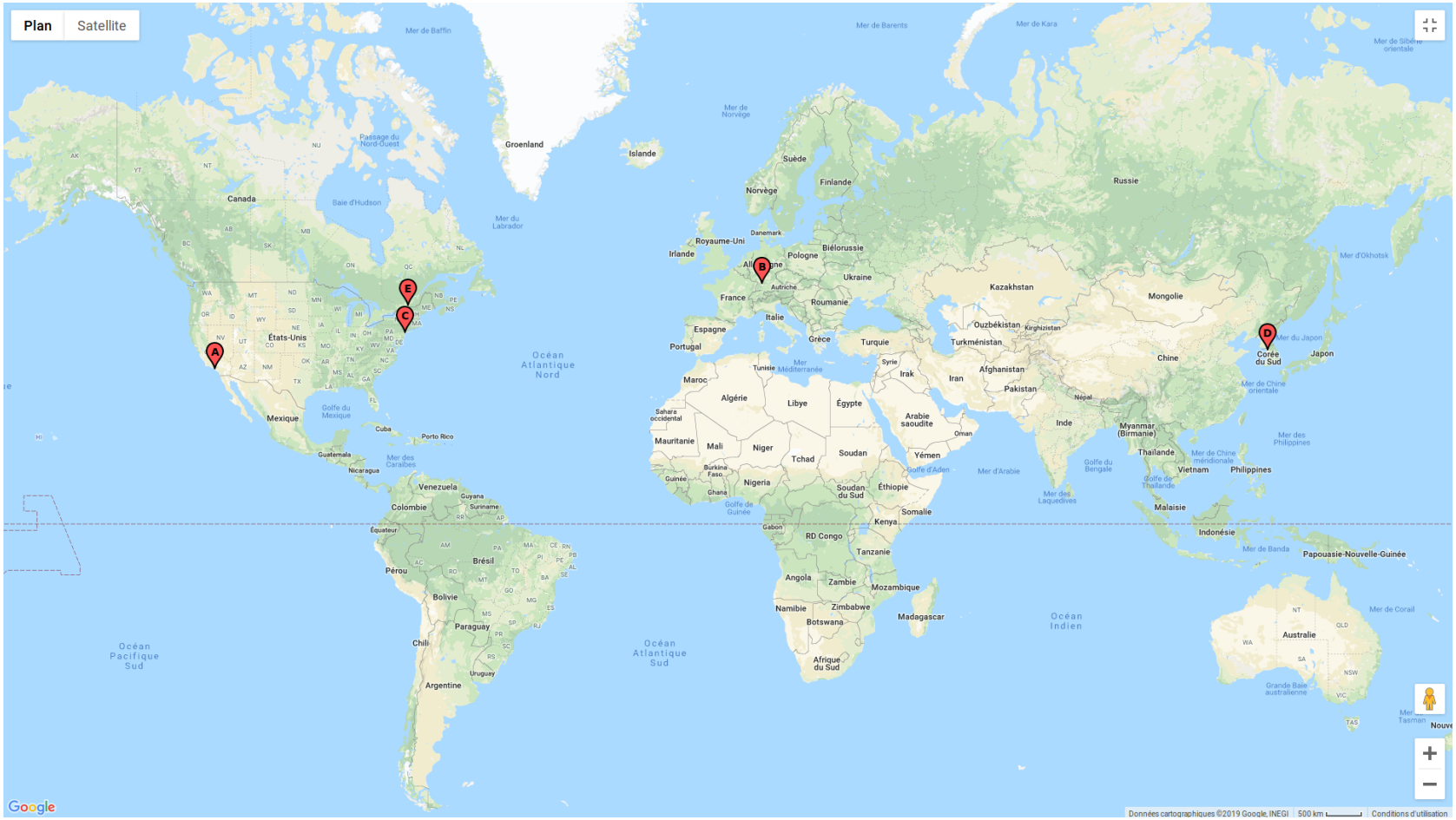
Why is it true?

- We expect car drivers to behave like fluids in a canal. However, they act *egoistically*.

Why is it true?

- We expect car drivers to behave like fluids in a canal. However, they act *egoistically*.
- In game theory, this is called *the price of anarchy.*

In real life...


Plan
Satellite
A
B
C
D
E
Canada
États-Unis
Mexique
Groenland
Islande
Norvège
Suède
Finlande
Russie
Royaume-Uni
Irlande
Pologne
Biélorussie
Ukraine
France
Espagne
Portugal
Italie
Grèce
Turquie
Roumanie
Kazakhstan
Mongolie
Chine
Japon
Corée du Sud
Inde
Pakistan
Afghanistan
Iran
Irak
Arabie saoudite
Égypte
Libye
Algérie
Maroc
Mauritanie
Mali
Niger
Tchad
Soudan
Nigeria
Éthiopie
Somalie
Kenya
RD Congo
Tanzanie
Angola
Zambie
Mozambique
Madagascar
Namibie
Botswana
Zimbabwe
Afrique du Sud
Brésil
Pérou
Bolivie
Paraguay
Chili
Argentine
Uruguay
Colombie
Venezuela
Guyana
Suriname
Cuba
Porto Rico
Australie
Indonésie
Papouasie-Nouvelle-Guinée
Océan Atlantique Nord
Océan Pacifique Sud
Océan Atlantique Sud
Océan Indien
Google
Données cartographiques ©2019 Google, INEGI
500 km
Conditions d'utilisation

In real life...

- Stuttgart, 1968

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- New York, for Earth Day, 1990

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- Seoul, Early 2000's
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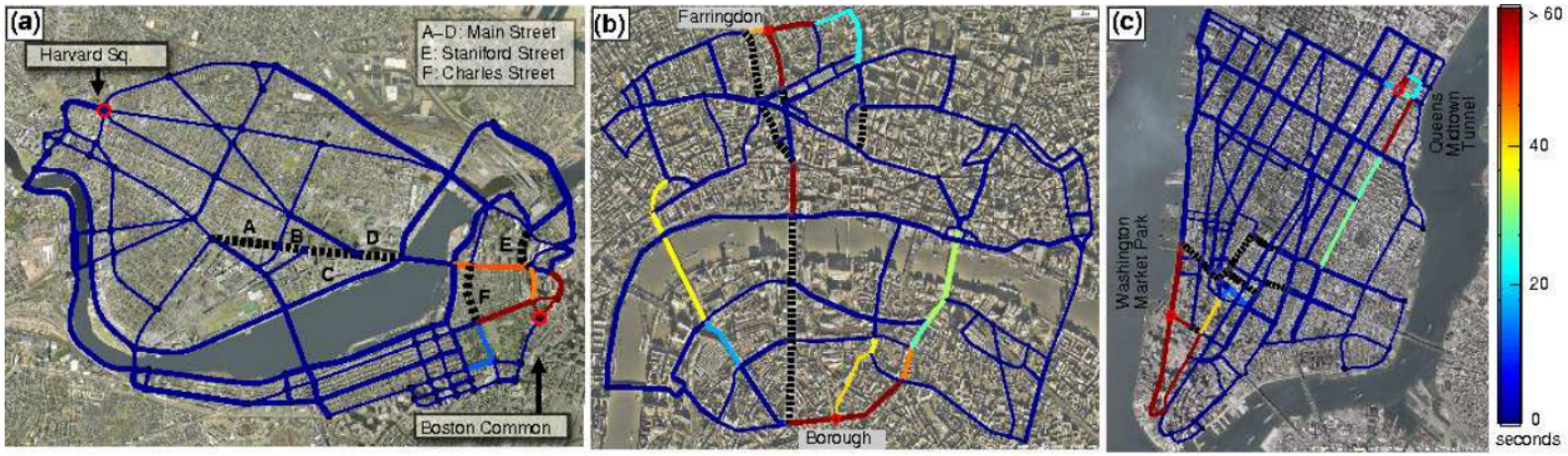
- Saint-Jean-sur Richelieu (?)

Can it happen again?

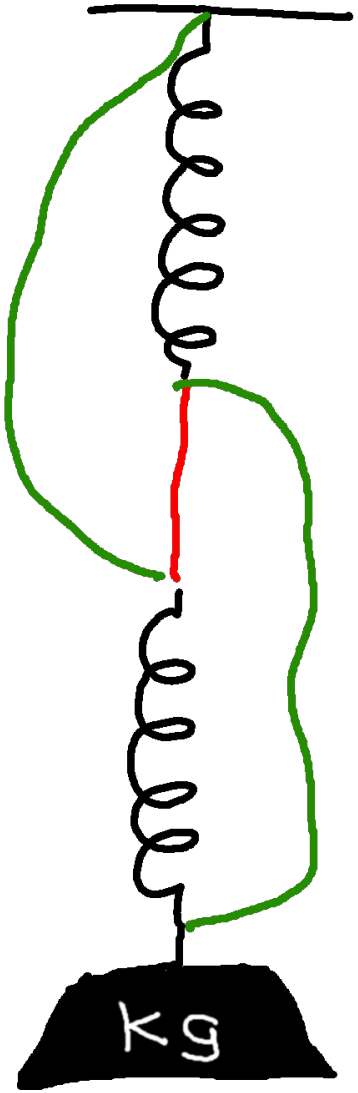
In 2008, two physicists and a computer scientist identified itineraries in Boston, London and New York City that would be shorter if we would remove roads.



A physical demonstration

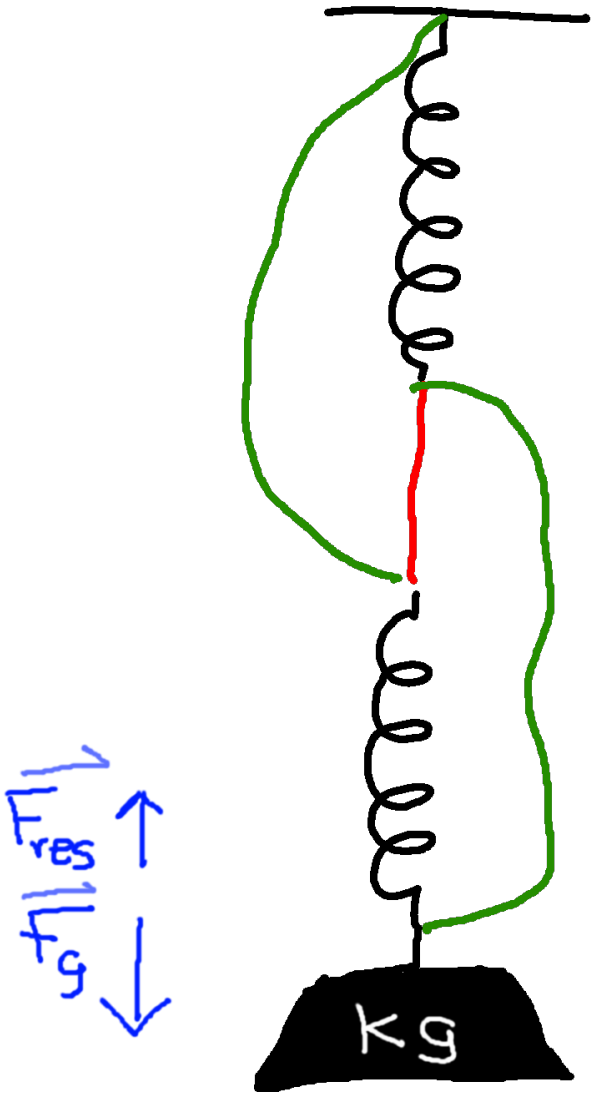
For the remaining skeptics...

How it works



What are the forces?

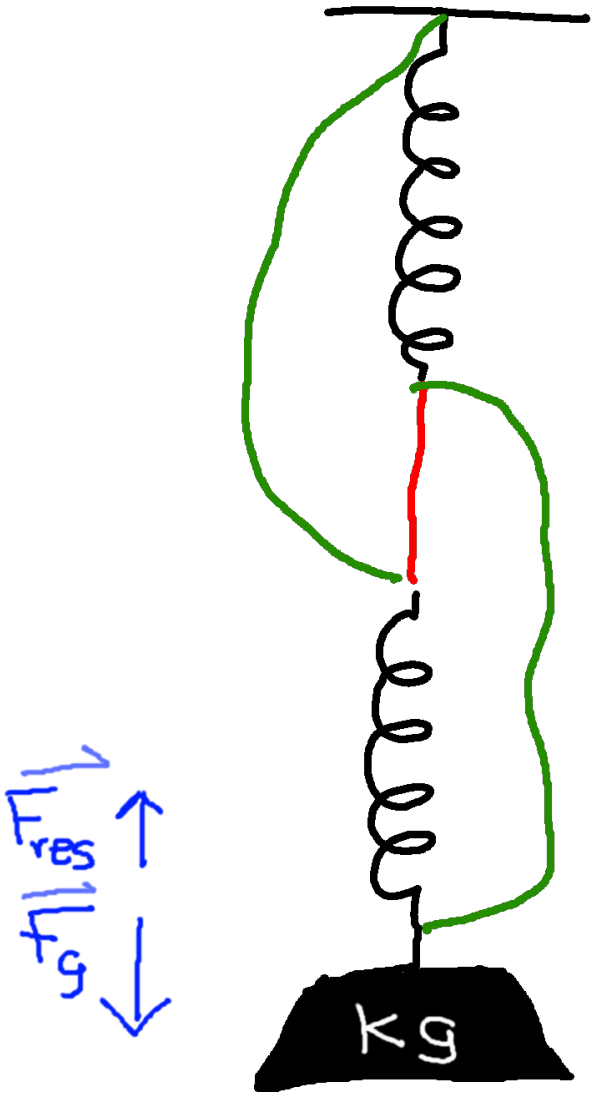
How it works



What are the forces?

- Gravity ($\vec{F}_g$)
- Restoring force ($\vec{F}_r$)

How it works



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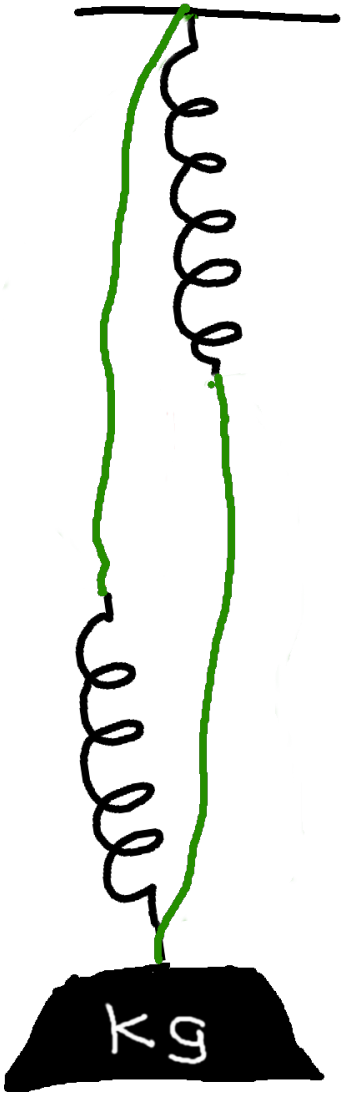
When balanced, Hooke's law says

$$-\text{weight}\cdot 9.8 m/s^2 = -\vec{F_g} = \vec{F_r} = k\cdot\Delta x.$$

How it works



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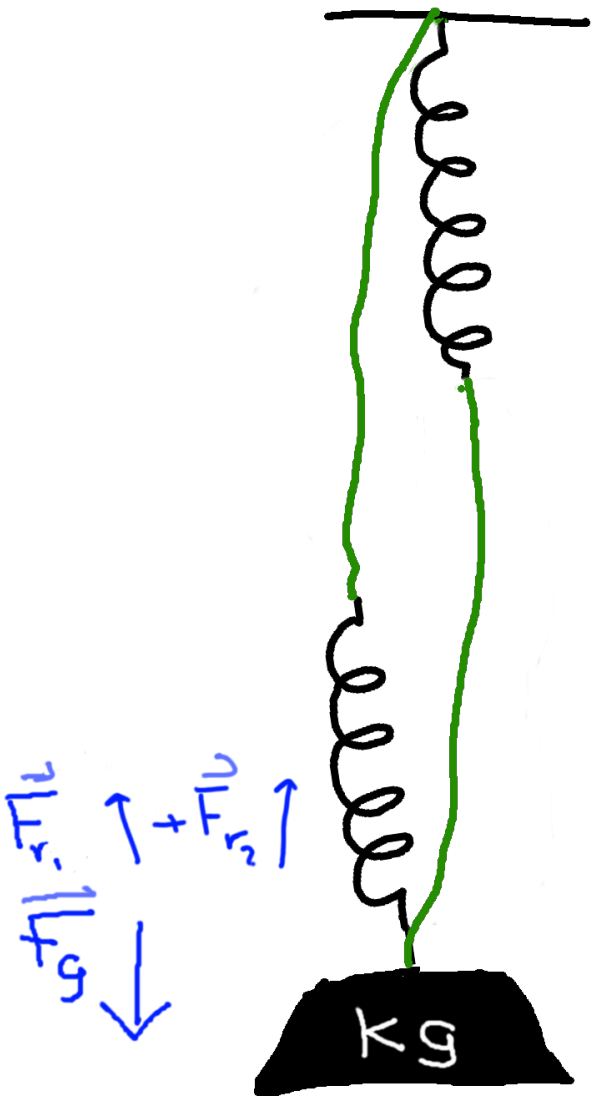

Restoring force is split into two:

$$\vec{F}_{r_1} = \vec{F}_{r_2} = \frac{-\vec{F}_g}{2}$$

and the stretching of the springs is half of what it was before.

Other occurrences

- Communication networks
- Basketball tournaments
- Electrical networks
- Study of extinct populations